A Fast Robust Surface Reconstruction Algorithm by Robust Ellipsoid Criterion and Down Sampling

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Abstract: Advanced 3D scanning technologies enable us to obtain dense and accurate surface sample point sets. From sufficiently dense sample point set, Crust algorithm, which is based on Voronoi diagram and its dual Delaunay triangulation, can reconstruct a triangle mesh that is topologically valid and convergent to the original surface. However, the algorithm is restricted in the practical application because of its long running time, and when the point cloud must not be noisy, the surface reconstructed is not good. Surfaces are often reconstructed from unorganized point sets with noise, so denoising is an essential step in creating perfect point-sampled models. A novel surface reconstruction algorithm is proposed. Firstly, this paper determines if one point is the noise or not by the ellipsoid criterion. After acquiring new point sets being less noisy, we smooth the remains noise by mean shift point clouds denoising method. Experiments show that our method can smooth the noise efficiently. Secondly, a non-uniformly sampling method is used to resample the input data set according to the local feature size before reconstruction. Finally, the surface is reconstructed by crust algorithm. In this way, the speed of reconstruction is increased for noisy points without losing the details we need.

1. Introduction

Reconstructing 3D surfaces from point samples is a well studied problem in computer graphics. It allows fitting of scanned data, filling of surface holes, and remeshing of existing models.

In recent years, there have been a series of papers to solve the original surface reconstruction problem using Delaunay-based approach without handling with the sharp feature. Hoppe et al.[1] highlighted the importance of addressing the surface reconstruction problem from only a set of sample points obtained from the original surface by using signed distance function and tangent planes to serve as local linear approximation to the surface followed by the contour tracing algorithm, to derive the original surface. Edelsbrunner and Mucke[2] designed the α-shape algorithm using a more refined sculpting strategy. Amenta, Bern and Kamvysselis[3] proposed the first algorithm (called crust) which provides theoretical guarantees on the reconstructed surface. Then Amenta et al.[4] designed a co-cone algorithm , which is more efficient and simpler than the original crust algorithm. Several more followup results using similar Delaunay-based approach are the power crust algorithm of Amenta et al.[5], the extended co-cone algorithms by Dey et al.[6] which detects under-sampled region and [7] which reconstruct the surface while leaving no holes, and the recent reconstruction algorithm of Yau et al.[8], which uses a robust and efficient region-growing algorithm to deal with the surface reconstruction problem. Another approach for reconstructing surfaces proceeds in the way to grow a triangulated surface by attaching more new triangles successively to the boundary of the current reconstructed surface[9] with different geometrical structures.

The implicit surface approach has also been used to reconstruct surfaces[10] that basically fits implicit surfaces to the input points with different criterions to minimize the energy that represent different distance functions. These algorithm represent the surface with a set of high degree functions instead of discrete triangles and their topology relation.

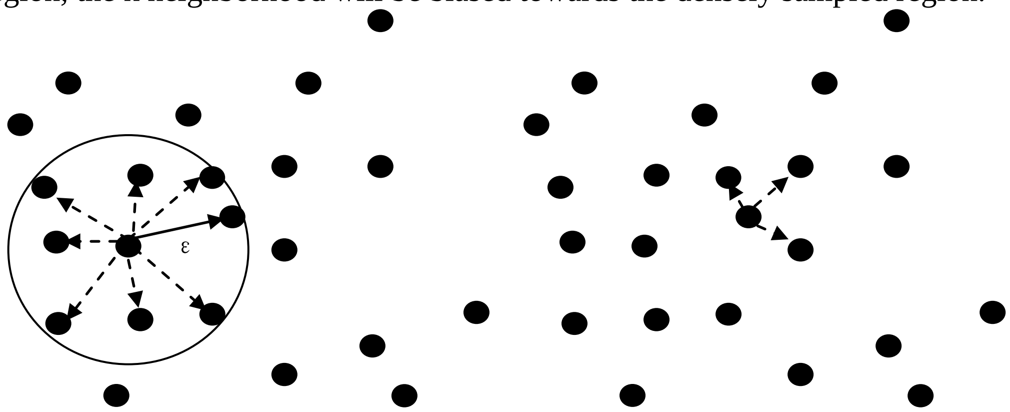
Compare to other algorithms, the Crust algorithm is not only simple and direct in theory but also faithful to the original surface. However, Crust algorithm is too slow for many practical applications with current computing resource, and it can not reconstruct noisy points effective.

In this paper, a point cloud denoising method which combines the ellipsoid criterion and mean shift filtering approach is proposed in this paper, which can handle the noise and readily be formulated for mesh-based geometry and even for general 3D geometry. Firstly, the data points will be pre-processed by ellipsoid criterion. For each data point, we determine if it is noise or not by the ellipsoid criterion. If the boundary probability is not ‘Interior’, this point is the noise. Most of the large-scale noise points can be detected in this step. Then, a non-uniform down sampling method as in [11] for large and unorganized point set is presented before surface reconstruction according to the local feature size. Finally, the point set is reconstructed by the method of Power Crust. We propose a new fast and effective method that not only give good reconstruction to the smooth areas of the given surface, but also recovers the sharp features originally existed in the source surface model provided that a reasonably nice sampling point cloud is given.

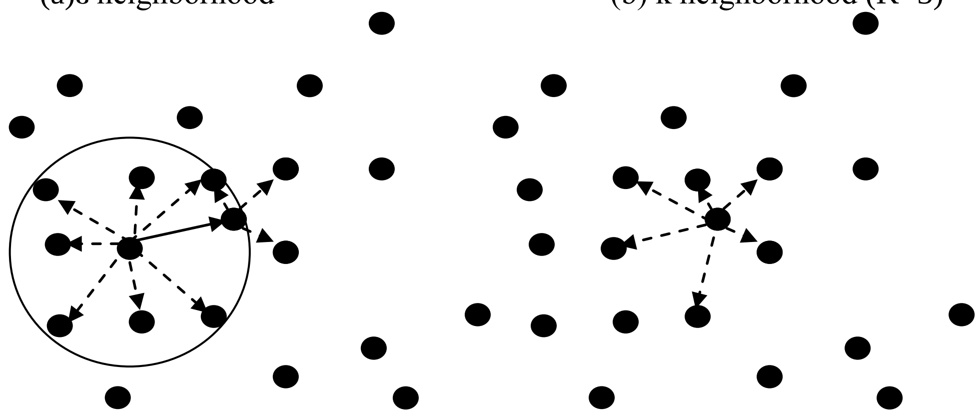
2. Noise Deleted

2.1. Neighborhood selection

A very common definition of local neighborhoods around a point p found in the literature is the k neighborhood $N^k(p)$, consisting of the k nearest samples in P to p. This simple definition, though, becomes unreliable in areas of varying sampling density. In points lying on the edge of a densely sampled region, the k neighborhood will be biased towards the densely sampled region.



(a)ε neighborhood (b) k neighborhood (K=3)



(c) $N^{k\varepsilon}$ and improved (d) $N^{k\varepsilon}$ neighborhood (K=6)

Fig.1. ε neighborhood, k neighborhood, $N^{k\varepsilon}$ and improved $N^{k\varepsilon}$ neighborhood

This problem can be alleviated somewhat by the $N^{k\varepsilon}(p)$ neighborhoods. It includes not only the k nearest points but also all points inside a small sphere with radius ε. By selecting an appropriate value for ε, the biasing effect can be reduced, but the neighborhood of points in densely sampled regions will contain more points than necessary, increasing the cost of evaluating the ellipsoid criteria, which effectively limits the range of a feasible ε.

To overcome the biasing effect, it therefore typically suffices to include these nearby points in the neighborhood.

$$\overline{N}_x = \left\{ y \in X \middle| y \in N_x^{k\varepsilon} \vee N_y^{k\varepsilon} \right\} \tag{1}$$

To complete the neighborhood for the critical points, we hence define that if point x is one of point y's neighbors, then point y is considered one of point x 's.

In Fig.1 (c) and (d), we can see that the neighborhood is unbiased to the unorganized point data.

Because of the noisy data points, we consider the extent neighborhood, depending on the noise level of the dataset. In this article, we compute the weighted average by the two layer neighbors.

2.2. Noise deleted ellipsoid criterion

Gumhold [12] uses the correlation matrix formed by the neighborhood. The eigenvectors and eigenvalues of this matrix define a correlation ellipsoid. Its shape, expressed in the ratios of the eigenvalues, is used to identify corner, crease and boundary points and also gives an approximation to crease and boundary direction. In order to find continuous crease lines, a neighborhood graph on the point set is built and its edges are weighted according to the crease probability.

As noted in [13], ellipsoid criterion of detecting shape probability performs best in the presence of noise. The wide spread k-nearest samples neighborhood, denoted as N(p). The shape of the correlation ellipsoid of approximates the general form of N(p) the neighboring points.

Where $C_i = \frac{1}{|N^{k\varepsilon}|} \sum_{p \in N^{k\varepsilon}} (p - \overline{p_i})(p - \overline{p_i})^t, C_i \in R^{3\times 3}$ is a symmetric weighted positive semi-define matrix, $\overline{p_i} = \frac{1}{|N^{k\varepsilon}|} \sum_{p \in N^{k\varepsilon}} p, \overline{p_i} \in R^3$ is the weighted average of N(pi). Parameters λ1, λ2, λ3 are the eigenvalue of the matrix, and unit vectore1, e2, e3 are corresponding eigenvector. We supposeλ1≤λ2≤λ3.

We collect the relative magnitudes of the eigenvalues in a vector $h_p = (\frac{\lambda_1}{\xi}, \frac{\lambda_2}{\xi}, \frac{\lambda_3}{\xi})$,with $\xi = \lambda_1 + \lambda_2 + \lambda_3$.There are five characteristic situations = {Boundary, Interior, Corner, Line, Ridge}.

In the case of interior points, the ellipsoid degenerates to a circle and $h_{\text{int}erior} = (0, \frac{1}{2}, \frac{1}{2})$. If points are on one hline=(0, 0,1). For boundary points, the ellipsoid becomes an ellipse in the tangent plane then $h_{boundary} = (0, \frac{1}{3}, \frac{2}{3})$. Finally, for points on the ridge, $h_{ridge} = (\frac{1}{4}, \frac{1}{4}, \frac{1}{2})$. The anterior three values of h span a triangle Δh containing all possible values for h. We now extract classification probabilities $\tilde{\tau}_t$ for each shape.

Nowτt is given as,

$$\tau_t = g_{\sigma t}\left(\left\| h_p - h_t \right\| \right) \tag{2}$$

In the article, we use a Gauss kernel Parameterσt is defined as,

$$\sigma_t = \frac{1}{2} \left\| h_t - centroid\,(\Delta_h) \right\| \tag{3}$$

Because the different shapes regions overlap. We define the final boundary probability $\tilde{\tau}_t$ as,

$$\tilde{\tau}_t = \frac{\tau_t(p)}{\sum_{t \in \{r,l,b,i,c\}} \tau_t(p)} \tag{4}$$

Our algorithm is performed in the following procedures:

Step1. Give the nearest point number k and the radius ε of a small sphere according to the point density.

Step2. Select one point p i from the point sets and then compute the center location $\overline{p_i}$ and the correlation matrix C I. At the same time we can easily gain the eigenvalues {λ1, λ2, λ3} and the eigenvectors {e1, e2, e3}.

Step3. Collect the relative magnitudes of the eigenvalues in a vector $h_p = (\frac{\lambda_1}{\xi}, \frac{\lambda_2}{\xi}, \frac{\lambda_3}{\xi})$, with $\xi = \lambda_1 + \lambda_2 + \lambda_3$. We computer the center of the triangle centroid (Δh).

Step4. For five characteristic situations t∈T = {Boundary, Interior, Corner, Line, Ridge}, we compute the Gauss kernel gσ , then extract classification probabilities $\tilde{\tau}_t$ for each of the shapes described above. If $\tilde{\tau}_{interior}$ is the maximum of $\tilde{\tau}_t$, t∈{Boundary, Interior, Corner, Line, Ridge}, the point is the interior. Otherwise, we define the point is the noise.

Step5. If there are points in P, go to Step 2.

3. Surface Reconstruction Algorithm

3.1. Power crust algorithm

We assume that the input point set S is a sufficiently dense sample of a smooth surface.

In Crust algorithm, we can see the main steps are computing the Voronoi diagram of the sample, selecting the poles in the Voronoi vertices to estimate the medial axis, then we compute the Delaunay triangulation of the combined point set of the samples and poles, in the end we choose the triangles whose vertices are all samples to output crust as surface. From the process of the algorithm, we can see that the most time-wasting step of Crust algorithm is the computation of 3D Voronoi Diagram and Delaunay triangulation. Notice that the number of sample and poles is at most 3n, the time complexity of the algorithm is about O(n2)+ O(9n2) , where n is the number of input points. Therefore, there are two ways to reduce the complexity: improve the efficiency of the computation of 3D Voronoi Diagram, or decrease the number of points. Voronoi diagram and its dual Delaunay triangulation have been studied widely since it was presented in 1936. It is difficult to improve efficiency of algorithm in advance. Thus we try the second way.

Notice that the local feature size is big in featureless area and small in detailed area, Crust does not require dense sample everywhere. However, as the surface is unknown, sample device can't know the local feature size of the area it is sampling, it is almost impossible to realize r-sample. If we do it manually, on the one hand the sampling process will be quite troublesome, on the other hand people can only evaluate how detail the surface is so that the sample can't be very well coincident to the r sample's requirement. In order to maintain the detail information in the reconstructed model, people usually desire the sample as dense as possible. The result is that the input point set is often with a great deal of points that are not necessary to correct reconstruction. If we discard these points, we can still correctly reconstruct the surface without losing details. In addition, the running time of reconstruction will be reduced.

3.2. Non-uniformly down sampling

If S is an r-sample of F and p is a point on F, then the distance between p and its nearest sample point s is within r · LFS(p). Since every sample is also a point on F, the distance between s and s1 is no more than r · LFS(s), where s1 is the nearest point of s in S.

As show in figure 1, s is a point in S, v is the negative pole of s, s1 is another point in S that d(s, s1) = rs · LFS(s). Let s be the center and rs · LFS(s) be the radius, we have the ball B1. Let v be the center, LFS(s) be the radius, we have another ball B2. In accordance with the definition of local feature size, s1 is outside ball B2. Passing through s we make a plane L tangent to F. Because of the assumption that the surface is smooth, s1 and B2 must be located the same side of L. From the above discussion, we can see that F must be in the shaded region of figure 1 if it is in B1.

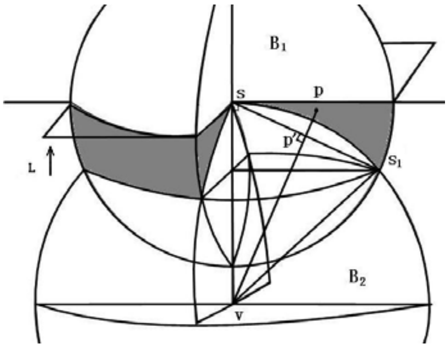


Fig.2. Down sampling

There are two factors influencing local feature size – the curvature and proximity of the other parts of the surface. However, the second factor can't affect the local feature size in a small region, so we need not take into account the factor in a local area. That is to say, the local feature size is inversely proportional to the curvature in the shaded region when r is small enough.

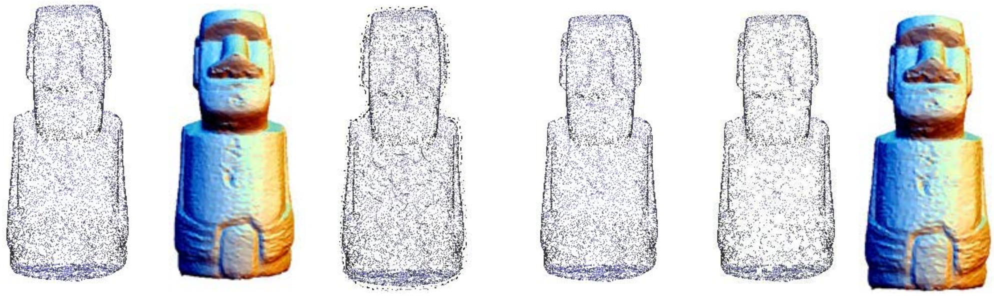
Let p be a point on the surface in the shaded region, and p' is the intersection of the line pv and B2. As we all known, the more flat the surface is, the lower the curvature is. It is apparent that the curvature of point p is smaller than that of point p'. Since point p' and s are both on the ball B2, their curvatures are the same. Thus, we have LFS(p) ≥ LFS(s) . In addition, on account of that p is in the shaded region, we have d(s, p) ≤ d(s, s1). As a result, we get d(s, p) ≤rs · LFS(s). As S satisfies the requirement for r-sample, rs is less than r. So, we have d(s, p) ≤r · LFS(p).

Then, we can make the following conclusion: if we can find another point s'∈S that satisfied equation d(s, s') ≤r · LFS(p), S is an r-sample of a surface F. Therefore, if we delete all the points in the shaded area excepting the farthest one and s itself, the downsampled point set S ' is still an r-sample of F, and an r-sample point set is sufficiently dense for correctly reconstruction if r is no more than 0.5. Thus, r should be less than 0.5 here. In fact, we obtain good result when r = 0.5.

Down sampling:
1) Initial every point in S as unmarked
2) for(i=0;i<n;i++){
3) if si is unmarked{
4) dmax=0;m=0
5) for(j=0;j<n;j++)
6) if sj is unmarked{
7) if d(si, sj) ≤ r· LFS(si){
8) marked sj;
9) if (d(si, sj)<dmax update dmax and m
10) }}
11) unmarked sm;
12) select all the unmarked points as the down sampled point set

4. Experimental Results and Analysis

We experiment with the Moai model. Experimental results from improved FCM method base noise deleted, down sampling and surface reconstruction are now investigated. Figure 2 illustrates the performance of approach in the paper.



(a) (b) (c) (d) (e) (f)

Fig.3. Noise deleted and down sampling of Moai model. (a)Original point clouds. (b)Original model. (c)Noisy point cloud, the number of the noise is 2500, and the number of the point is 10002. (d)Noise deleted by our Robust ellipsoid criterio. (e)Down sampling by our methd, the number of the point preserved is 3350. (f)The output of our algorithm running on the Moai model

Just as our expectation, the density of down sampled point set is varied according to the surface's detail. The samples are still very dense in the region like the eyes and nose of Moai. But in the featureless region, such as the body, it is very sparse compare with the original dataset. In the example of Stanford bunny the points are reduced relatively uniformly. It is because that the surface of bunny does not change very quickly.

Form the result we also can see that the reduction of data is varied with the different dataset. It is relied on the density of the input points: the denser the input data set is, the more points we can delete. In the example of Moai, the size of new data set is reduced to about 1/3.

5. Conclusion

We have presented a improved method of Robust ellipsoid criterio and non-uniform down sampling method for dense and unorganized noisy point set before surface reconstruction according to the local feature size. Guaranteeing the topological shape, we use a smaller point set to reconstruct the noisy point cloud. As the result, noisy point set can be reconstructed effectively and the speed is improved. This method also can be applied in mesh simplification. In fact we can use r-sample to define the level of detail for mesh. With the increasing of r, the mesh's level of detail is decreasing. So we can realize mesh simplification by using this down sampling method to build an r-sample model with bigger r.

As the triangles in the plat areas are relatively large, the whole model looks coarse. However, Gouraud shading can give us a tolerable visual effect when r is not very big. In addition, if we want a more elaborate visual effect, subdivision can be used to get smooth surface.

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